Enhancing Geometry Teaching in STEAM Education with Interactive Learning Environments

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Manuscript received January 12, 2024; revised February 15, 2024; accepted February 21, 2024; published September 14, 2024

***Abstract*—In confronting the challenge of effectively integrating technology into mathematics education, this study investigates the impact of GeoGebra, an interactive learning tool, on enhancing the understanding of geometry among high school students. Through a rigorous mixed-methods approach, involving 150 students segmented into control and experimental cohorts, this research meticulously assesses the differential outcomes attributable to GeoGebra engagement. Notably, our findings illuminate a significant augmentation in the experimental group's grasp of geometric concepts, with a pronounced improvement in correlating algebraic and graphical representations of trigonometric functions. This advancement was not merely academic, it corresponded with elevated levels of student engagement and performance. Beyond the anticipated benefits of interactive tools in educational settings, our study contributes novel insights into the specific advantages of GeoGebra within the STEAM curriculum. The results advocate for a more pronounced integration of technology-enhanced learning environments in educational systems, aiming to not only enrich students' learning experiences but also to prepare them more effectively for the complexities of modern scientific and mathematical problem-solving.**

***Keywords*—education, geometry, interactive learning environment, mathematics, pedagogics**

I. Introduction

In the realm of STEAM education, the integration of technological advancements offers a transformative avenue for enhancing geometry learning [1]. The initial foray into this discussion acknowledges the pivotal role of interactive tools, such as GeoGebra and Augmented Reality (AR) [2], in addressing the perennial challenges of geometry education—chief among them, the abstract nature of its concepts and the difficulty students face in spatial visualization [3]. These challenges, well-documented in academic literature, underscore the need for pedagogical strategies that transcend traditional teaching methods, which often fail to fully engage students or foster a deep understanding of geometric principles [4–6].

The literature review reveals a burgeoning interest in the application of technology to facilitate a more immersive and intuitive learning experience in mathematics education [7]. Despite the promising outcomes of preliminary studies, there remains a conspicuous gap in comprehensive, empirically grounded research into the effectiveness of these technologies in real-world classroom settings [8]. Specifically, the literature points to a dearth of studies examining the long-term impact of interactive learning environments on student achievement and engagement in geometry, within the broader context of STEAM curricula [9].

Addressing this gap, the study's primary purpose is to rigorously evaluate how the implementation of GeoGebra and AR can transform students' understanding of geometry, focusing on high school students. The present study is predicated on the hypothesis that the use of GeoGebra and AR can significantly enhance high school students' understanding of geometry, not only in terms of academic performance but also in fostering greater engagement and applicability of geometric concepts in real-world scenarios. This hypothesis is rooted in constructivist learning theories [10], which advocate for the role of active engagement and experiential learning in the cognitive development and retention of complex information [11].

The methodology outlined for this investigation aims to rigorously assess the impact of these interactive tools through a mixed-methods approach, encompassing both quantitative and qualitative analyses [12–14]. This comprehensive strategy is designed to provide a holistic view of the educational value of GeoGebra and AR in geometry learning, encompassing a diverse range of learning outcomes [15].

The identified literature gaps are twofold: firstly, there is a noticeable absence of long-term studies assessing the enduring effects of technology-enhanced learning tools on students' understanding and engagement in geometry. Such an oversight limits our comprehension of the sustained academic and motivational impacts of these educational technologies. Secondly, the comparative effectiveness of different technological tools, such as GeoGebra and Augmented Reality (AR), in facilitating geometry learning is insufficiently explored. This study endeavors to address these gaps by evaluating both the longitudinal benefits of these tools in educational settings and their relative efficacy in improving students' geometric comprehension, thus contributing novel insights to the field of STEAM education.

In summary, the significance of this study extends beyond the immediate academic performance metrics, offering insights into the pedagogical benefits of integrating technology into geometry education. By contributing to the body of evidence supporting the use of interactive tools in enhancing student learning, this research holds the potential to inform future educational practices and policies, ensuring that geometry education is both engaging and effective in preparing students for the complexities of the modern world. The paper is structured to take the reader on a journey from the identification of the problem and literature review, through the methodological approach, to the presentation of

doi: 10.18178/ijiet.2024.14.9.2153

findings and their implications for educational practice and policy.

This study posits several hypotheses to examine the impact of interactive learning environments on geometry education within the STEAM framework.

Hypothesis I: Students who engage with interactive learning tools in geometry lessons will demonstrate significantly higher levels of understanding and application of geometric concepts compared to students who receive traditional instruction.

Hypothesis II: The use of interactive learning environments in geometry education will lead to increased student engagement and motivation in learning STEAM subjects, as measured by attendance, participation in class activities, and students' self-reported interest levels.

II. Related Works

This section of the paper delves into various studies and developments pertinent to the integration of Augmented Reality (AR) in geometry education. It aims to contextualize the current research within the broader scope of technological advancements in educational settings, particularly focusing on the transformative impact of AR in learning processes.

A. Early Developments in AR for Education

The inception of Augmented Reality (AR) technology in educational contexts marked a pivotal shift in teaching and learning paradigms. Initially conceptualized as a means to enhance real-world environments with digital information, AR's early applications in education were rudimentary yet groundbreaking. The primary focus during these nascent stages was to overlay digital images or data onto physical textbooks or classroom environments, providing a novel sensory experience to learners [16].

This early exploration into AR was characterized by its simplicity. The overlays were basic, often consisting of static images or text that would appear when viewed through AR-enabled devices. Despite their simplicity, these initial implementations sparked a curiosity and interest in further exploring the potential of AR in educational settings [17]. It was recognized early on that AR held a unique capacity to bridge the gap between abstract theoretical concepts and tangible, experiential learning.

As technology advanced, so too did the capabilities of AR tools. What began as simple overlays soon transformed into more sophisticated, interactive systems. These advancements were propelled by improvements in hardware, such as more powerful mobile devices, and software, including more intuitive and accessible AR platforms [18]. The evolution of AR tools witnessed a transition from passive to interactive learning experiences. Students could now engage with educational content in a dynamic way, manipulating digital objects and participating in immersive simulations [19].

This progression in AR technology opened up a plethora of possibilities for its application in education. AR tools evolved to include features like 3D modeling, real-time interaction, and gamification elements, which were particularly effective in subjects requiring spatial understanding and visualization, such as geometry and science [20]. The interactive nature of these advanced AR tools not only made learning more engaging but also catered to various learning styles, accommodating visual, auditory, and kinesthetic learners alike [21].

Furthermore, the development of AR in education paralleled broader pedagogical shifts towards more student-centered and experiential learning approaches. The ability of AR to create immersive learning experiences aligned well with educational strategies that emphasized active participation and learning through doing [22]. As educators and researchers recognized the potential of AR to enhance learning outcomes, there was a notable increase in the integration of AR tools in educational curricula and an expansion in the variety of subjects where AR was applied [23].

B. AR in Mathematics and Geometry Education

The integration of Augmented Reality (AR) into mathematics and geometry education signifies a transformative step in the teaching and learning of these disciplines. Numerous studies have delved into the efficacy of AR tools in this context, revealing substantial improvements in students' learning experiences and outcomes [24]. The unique capability of AR to merge digital information with the physical world offers an enhanced approach to understanding abstract mathematical concepts, particularly those requiring spatial reasoning and visualization.

In the realm of geometry education, AR tools have been shown to facilitate a deeper comprehension of geometric shapes and properties. These tools allow students to visualize and interact with three-dimensional shapes, a critical aspect often missing in traditional two-dimensional teaching methods [25]. The interactive nature of AR aids in bridging the gap between theoretical understanding and practical application, making abstract geometric concepts more tangible and relatable for students.

Studies focusing on AR in mathematics education have highlighted significant gains in student engagement and motivation [26]. The novelty and interactivity of AR applications capture students' attention, fostering an environment conducive to active learning. This engagement is further enhanced by the hands-on experience that AR provides, allowing students to manipulate mathematical models and explore geometric relationships in a dynamic and intuitive manner [27].

Furthermore, research has shown that AR tools can significantly enhance spatial reasoning and visualization skills in students. These skills are fundamental in geometry and are often challenging to develop through conventional teaching methods [28]. AR's capability to present spatial information in a more accessible and interactive format has been found to aid in the development of these skills, leading to a more robust understanding of geometric concepts [29].

In addition to enhancing understanding and skills, AR in geometry education also opens up opportunities for differentiated instruction. By allowing customization of learning experiences, AR can cater to varying learning styles and abilities, making geometry education more inclusive and effective [30].

C. Pedagogical Theories Supporting AR in Education

The integration of Augmented Reality (AR) in education is strongly supported by modern pedagogical theories, particularly constructivism and experiential learning. These theories advocate for a learning process where students construct knowledge through experiences and interactions within their environment [24]. AR, with its immersive and interactive capabilities, aligns seamlessly with these educational philosophies.

Constructivist theory posits that learning is an active, contextualized process of constructing knowledge rather than passively acquiring it [25]. AR provides a platform where learners can engage directly with educational content, thereby facilitating the construction of knowledge through interactive experiences. This active involvement is crucial in subjects like geometry, where understanding is deepened through practical, hands-on interaction with geometric shapes and concepts.

Experiential learning theory, similarly, emphasizes the importance of experiences in the learning process [26]. AR tools offer rich, experiential learning environments where abstract concepts can be visualized and manipulated in real-time. This not only aids in comprehension but also allows students to apply theoretical knowledge in practical scenarios, enhancing their learning experience.

Moreover, AR tools are aligned with the shift towards more student-centered learning approaches in modern education. They encourage active participation, problem-solving, and self-guided exploration, all of which are key elements of effective learning strategies [27]. The ability of AR to provide personalized learning experiences caters to diverse learning styles and needs, making education more inclusive and accessible [28].

D. Comparative Studies on Traditional vs. AR-Enhanced Learning

The contrast between traditional teaching methods and AR-enhanced learning approaches in geometry education has been the subject of numerous comparative studies. These studies aim to delineate the differences in effectiveness, engagement, and comprehension between conventional classroom methods and innovative AR-based techniques [30].

Research comparing these two approaches consistently demonstrates the superiority of AR-enhanced learning in several key areas. One significant finding is the heightened level of student engagement and interest when AR tools are employed [31]. This increased engagement is attributed to the interactive and immersive nature of AR, which transforms the learning experience from passive reception to active participation.

Another critical area where AR-enhanced methods outperform traditional ones is in the understanding and retention of complex geometric concepts. AR's ability to provide three-dimensional, interactive visualizations helps students grasp abstract concepts more effectively than two-dimensional representations typically used in traditional methods [32]. This enhanced understanding is especially evident in spatial reasoning and geometry problem-solving skills [33].

Furthermore, studies have indicated that AR tools can lead to higher motivation and better academic performance in geometry. The novel and engaging format of AR stimulates curiosity and encourages students to delve deeper into the subject matter [34]. Additionally, the personalized learning experiences offered by AR can cater to different learning styles, further enhancing its effectiveness compared to traditional methods [35].

III. Application of GeoGebra for Teaching Geometry

A. The Practice of Using GeoGebra Augmented Reality Technology

The GeoGebra Dynamic Mathematics system, a multifaceted educational tool, offers a range of instructional examples on its YouTube channel, tailored for enhancing mathematical learning. GeoGebra, recognized globally for its contributions to STEM education and innovative teaching methodologies, integrates geometry, algebra, spreadsheets, graphing, statistics, and calculus into an intuitive and free platform suitable for learners at various educational levels.

GeoGebra's software suite, compatible with various operating systems including mobile and personal computers, stands out for its accessibility and interactive capabilities. One notable application within this suite is the 3D Graphics program, specifically designed for mobile devices, currently available exclusively for iOS-operated Apple devices [36]. This program features functionalities for creating and manipulating 3D graphs, surfaces, and geometric shapes. Users can place these 3D mathematical objects in diverse environments like tabletops or floors, adjusting their size for enhanced visualization. Such features allow learners to observe mathematical concepts actively in their immediate surroundings.

Augmented reality (AR) plays a crucial role in GeoGebra's functionality, enabling users to construct polyhedra, view rotating geometrical figures, and experience these in a three-dimensional format [37]. The development of GeoGebra's AR component reflects an exploration into the educational potential of augmented reality in mathematics instruction. Currently, the application is in a developmental phase, with ongoing enhancements including the introduction of novel 3D shapes for more detailed examination and interaction. Fig. 1 demonstrates the surface in the GeoGebra augmented reality tool in use.

Experimentation with the AR features of GeoGebra can lead to various educational explorations. Users can input surface equations to observe their graphical representations or modify parameters to see real-time changes. The application also allows for the scanning of real-world objects to create corresponding mathematical models for further study, preparing users for advanced applications in their learning journey.

Tim Brzezinski, an avid user of the 3D Graphics AR application, has contributed significantly to its educational application [38]. His observations suggest that AR is a potent tool for exploring mathematical concepts and assessing student understanding. Brzezinski has created a GeoGebraBook, tailored for both educators and students, featuring lesson ideas and methods for employing the GeoGebra 3D Grapher in dynamic, student-centered learning environments.

Further highlighting GeoGebra's educational value, a workshop conducted by various researchers focused on educating and retraining mathematics teachers, incorporating supplementary student activities. This workshop demonstrated GeoGebra's effectiveness in STEAM education, emphasizing its role in mathematical art and augmented reality applications. The authors of this workshop provided an overview of the AR capabilities within GeoGebra, offering practical examples of its educational applications.

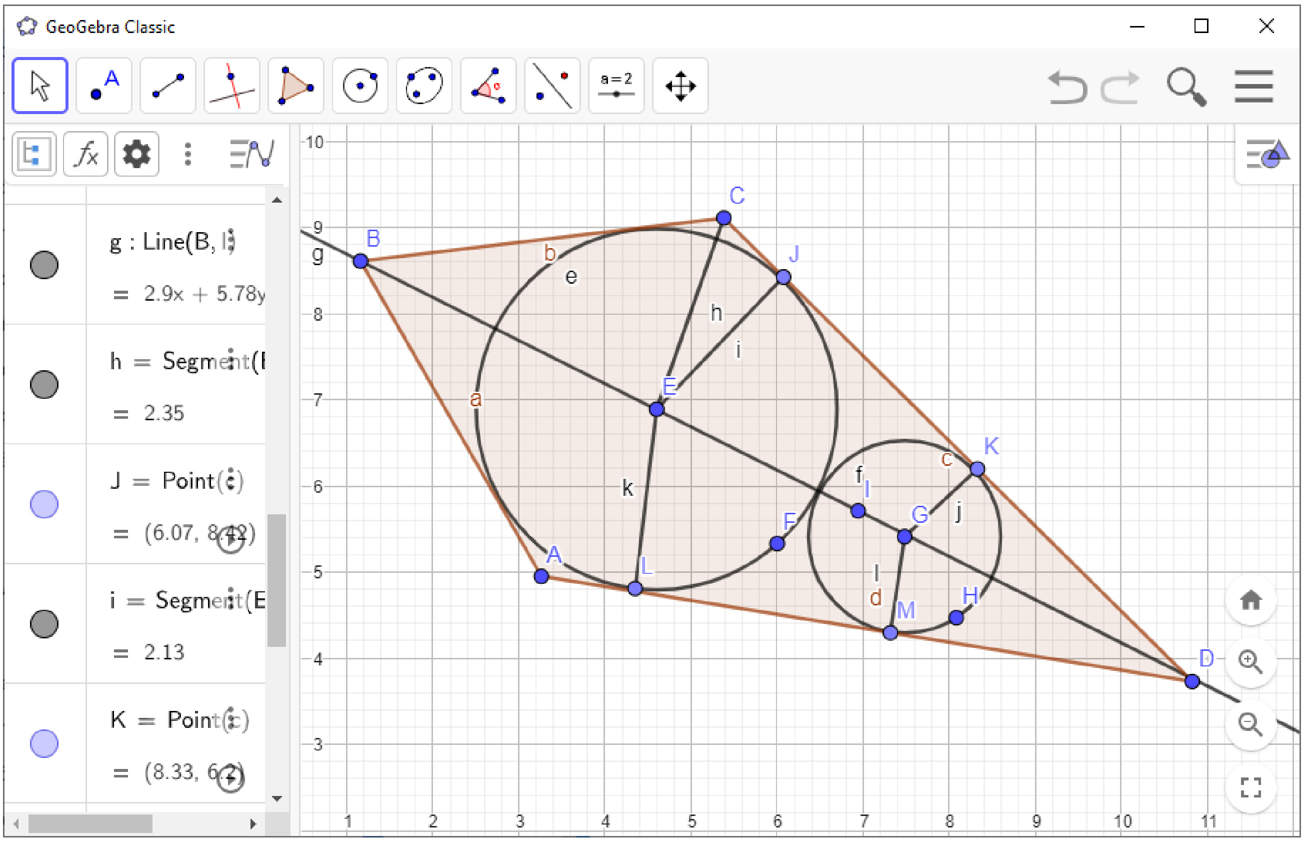
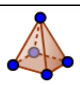
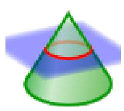
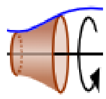
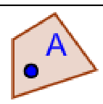
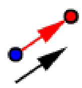
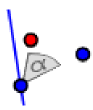


Fig. 1. The surface in Geogebra augmented reality tool in use.

B. Development of Visibility Tools with GeoGebra and Augmented Reality

To create a mathematical model using an Augmented Reality (AR) application, one must first design a 3D model using appropriate tools, as highlighted in Table 1, which lists tools compatible with the GeoGebra AR app. To place an object into a real-world setting, select a location, point the phone's camera towards it, and tap the screen. This action anchors the 3D figure at the chosen spot. Users can then interact with the model via the touchscreen, adjusting features like size and color.

Table 1. Course program that was developed integrating dynamic learning environment

Icon	Tool	Application
	Cube	To generate points that will establish the rib of the cube, double-click anywhere inside the 3D display.
	Pyramid	Double click to the view to generate Pyramid.
	Intersect two surfaces	Regardless of whether the surfaces have been given implicitly or in parametric form, the intersection of two algebraic surfaces is an algebraic space curve.
	Surface of revolution	A surface created by rotating a two-dimensional curve around an axis is known as a surface of revolution.
	Point of Object	Point in the object.
	Move	The "Move" enables to move items, and the first clicking on the object changes its location in the xOy plane, and the second clicking will alter the coordinate in the z axis.
	Vector from Point	A Vector has both magnitude and direction, but no fixed position in space.
	Rotate around Line	Every point on a line that has been rotated about its axis line will always be at the same distance from it.

One notable capability of the app is its function for constructing prisms, which can present unique challenges, especially when dealing with specific types like straight prisms with rectangular bases. To bring such a prism into an AR setting using the GeoGebra 3D Calculator, one must complete the prism's construction and then press the "AR" button. The next step involves using the camera to choose the real-world environment where the object will be placed. As illustrated in the table, tapping on the screen brings the object from the virtual realm into the physical world, where it can be explored more tangibly.

The smartphone camera effectively replaces our vision, allowing us to immerse in and view the virtual model from within. This immersion is enhanced by the app's features, which enable users to alter the model's scale and color. The AR functionality of GeoGebra reveals the ubiquity of mathematical shapes and objects in our surroundings. Users have the opportunity to explore these shapes by walking around them, looking inside, or even stepping into a model.

Considering the capability of the prism function in automatically calculating the volume of the created geometric shape, it becomes possible to explore the relationship between the actual volume of the shape and the calculated result given by the software. This exploration requires students to have the appropriate tools to investigate the properties of these shapes.

Previously, when introducing the concept of "Body Volume" to students, both traditional formula-based methods and a STEM-oriented approach were employed. The educational strategy involved encouraging students to formulate hypotheses regarding the volumetric relationships between various geometric shapes, such as prisms and pyramids, as well as cylinders and cones.

To facilitate this learning process, physical models of these shapes were constructed. The hands-on activity involved transferring dry materials from one shape to another–from a cone to a cylinder and from a prism to a pyramid. This practical approach allowed students to visually and tangibly grasp the concepts of volume and spatial relationships in a more engaging and interactive manner. Such activities bridge theoretical mathematical concepts with real-world applications, enhancing the understanding and retention of mathematical principles among learners.

IV. Methodology

A. Participants and Sample Size

For the purpose of this study, a carefully selected sample of 150 high school students, all of whom were scheduled to attend a STEM course at M. Auezov South Kazakhstan State University in the autumn semester of the 2022–2023 academic year, was utilized. From this group, 50 individuals were selectively assigned to the experimental Groups A and B, while the remaining 50 students were allocated to the control group. These participants were drawn from a variety of schools in the South Kazakhstan region, with the criterion for selection being their outstanding academic performance in regular classes during the 2022/23 academic year. In this study, the Simple Random Sampling method was used to select 150 high school students. This method ensures that every individual within the target population has an equal chance of being selected, thereby minimizing selection bias and enhancing the generalizability of the findings to the broader population of high school students. The choice of simple random sampling was driven by the aim to achieve a representative sample that could accurately reflect the diverse experiences and outcomes of students engaged with interactive learning environments in their geometry education. This strategic selection aimed to ensure a representative and high-performing cohort for the pedagogical experiment.

B. Research Methodology and Design

This research employed a blended methodology, combining experimental and observational techniques, alongside a non-equivalent control group design. This design included two experimental groups and one control group, facilitating a comprehensive evaluation of the instructional strategy's impact on student achievement [39]. A quasi-experimental approach was also utilized, with non-equivalent control groups assessed at the outset, midpoint, and conclusion of the experiment.

As delineated in Table 2, distinct methodologies were adopted for each of the three groups. The first experimental group (X1) engaged with the GeoGebra augmented reality tool for independent learning. The second group (X2) utilized GeoGebra under teacher guidance, while the control group (X3) continued with conventional teaching methods. These interventions were executed within the designated treatment timeframe.

The study's observational component involved an in-depth classroom analysis, employing the "seeing as" technique. This technique is a methodological approach that underscores the interplay between visual observation and cognitive understanding, facilitated through a diagnostic test [40]. This approach to classroom observation, whether executed in a formal or informal setting, aims to provide insights into instructional dynamics and student engagement within the learning environment.

Table 2. Pedagogical experiment with experimental and control groups

Experimental / Control Groups	**Pre-test results**	**Evaluation at the intermediate level and classroom observation**	**Post-test results**
Experimental Group A	O_1	T_1+X_1	O_2
Experimental Group B	O_1	T_1+X_1	O_2
Control Group	O_1	T_1+X_1	O_2

C. Experimental and Control Group

This research, spanning a semester, incorporated both experimental and control groups. The study was divided into two experimental groups, labeled Group A and Group B, alongside a control group. In Group B, comprising 50 students, the instructional approach involved introducing trigonometric functions through the use of GeoGebra. Students in this group collaborated on problem-solving and comprehending the material, including joint study sessions and exam preparation. Group A, also consisting of 50 students, engaged in collaborative learning during class hours, primarily through the exchange and discussion of GeoGebra files. The control group, with an equivalent number of 50 students, received instruction through traditional teaching methods.

D. Procedure

In alignment with the Grade 11 Kazakhstani mathematics curriculum, a comprehensive course spanning fifteen weeks has been structured. This course encompasses a series of twelve principal tasks based on GeoGebra, supplemented with additional exercises to reinforce the learning objectives. Subsequent to this, a range of activities specifically tailored for the experimental groups was devised utilizing the GeoGebra software. These activities, designed to enhance engagement and understanding, focus on making the subject matter more interactive, tangible, measurable, and visually accessible.

The inaugural session of this course featured an introductory overview of the GeoGebra program. This introduction was provided in the first hour of the lesson, setting the stage for subsequent sessions. Contrary to this initial session, in all following sessions, students engaged in GeoGebra-designed exercises that incorporated both visual and dynamic aspects, enriching the learning experience.

Moreover, the course integrated augmented reality tools to create textbook examples and illustrations, further enhancing the educational content. This innovative approach was employed throughout the sessions. The course, which requires a total of 80 hours of instruction, covers subtopics such as stereometry and vectors. It is structured around three distinct learning objectives, each tailored to maximize comprehension and application of these complex mathematical concepts.

E. Achievement Test

In this geometry learning study, the researchers developed eight test items. The reliability of these items, as measured by their internal consistency, was found to be 0.654. This reliability score is considered satisfactory [41]. A comprehensive achievement test was constructed, consisting of fifteen multiple-choice questions per lesson, coupled with graphical analyses of the topics covered. This test also included rationales derived from student textbooks, which were the primary resource for test preparation. The aim of this achievement test was to assess the effectiveness of the instructional strategies across the three student groups, in relation to the predefined objectives of the research. The test, crafted by the researchers, underwent review by two mathematics educators and was pilot-tested with 12th-grade students. Spanning a 15-week semester, the pilot test's main goal was to identify student comprehension challenges within the test activities and to develop open-ended inquiries for the principal study.

The research employed Cronbach's alpha [42] as a measure of internal consistency for the test items, with values ranging from 0 to 1, where higher values indicate greater reliability. An alpha value of 0.70 or above is generally accepted as indicative of good internal consistency.

Data analysis involved the use of t-tests for independent samples and one-way analysis of variance, applicable when data follows a normal distribution. A thematic analysis was conducted on observational data. In mathematics education research, t-tests are frequently utilized, particularly advantageous in studies with two distinct groups and limited samples [43]. Data analysis was conducted using Python 3.0, employing libraries such as Pandas, NumPy, SciPy, and Matplotlib.

V. Results

This research presents its findings in alignment with the formulated research questions. A key question addressed was the identification of any significant differences in the outcomes of pre-tests, intermediate-tests, and post-tests across all participating groups, encompassing both control and experimental cohorts.

A. Hypothesis Testing

To evaluate Hypothesis 1, which posits that students using interactive learning tools like GeoGebra in geometry lessons will show significantly higher levels of understanding and application of geometric concepts than those receiving traditional instruction, an independent samples t-test is employed. This statistical test is well-suited for comparing the means of two independent groups to determine if there is a statistically significant difference between them.

In this specific application, the independent samples t-test is used to compare the control task results of two distinct groups: Experimental Group A, which engaged with the interactive learning tool GeoGebra, and Control Group, which followed traditional instructional methods. The primary focus of this comparison is to assess whether the mean scores of the two groups on the control task, presumably a standardized test or assessment measuring understanding and application of geometric concepts, are significantly different.

The assumptions underlying this test include the independence of the two groups, the normal distribution of the dependent variable (control task results) in both groups, and homogeneity of variances. These assumptions ensure the validity of the test results. If the test yields a statistically significant result, it would support Hypothesis 1, indicating that the use of interactive learning tools like GeoGebra has a positive impact on students' understanding and application of geometric concepts in comparison to traditional teaching methods.

The independent samples t-test results presented for the control task scores of Experimental Group A and Control Group provide valuable insights for Hypothesis 1 in this research. Table 3 demonstrates the group statistics and the results of students that participated in the experiments. Table 4 demonstrates results of independent samples t-test.

Table 3. Group statistics for Hypothesis I

Group	N	Mean	Std. Deviation	Std. Error Mean
Experimental Group A	50	7.42	1.56609	0.22148
Control Group	50	4.5	1.97174	0.27885

Group Statistics Summary: Experimental Group A, which utilized the interactive learning tool GeoGebra, had a mean score of 7.4200 with a standard deviation of 1.56609. In contrast, Control Group, which received traditional instruction, had a lower mean score of 4.5000, with a higher standard deviation of 1.97174. The standard error mean for Experimental Group A was 0.22148, while it was 0.27885 for Control Group. This initial comparison suggests a higher average performance in the experimental group.

Independent Samples T-Test Analysis: Levene's Test for

Equality of Variances yielded an F-value of 1.940 and a significance (Sig.) value of 0.167, indicating no significant difference in variances between the two groups. Therefore, the assumption of equal variances is upheld.

The t-test for Equality of Means shows a t-value of 8.2. This is a crucial part of the analysis, as it reflects the magnitude of the difference between the two group means relative to the variation within the groups. With 98 degrees of freedom, the significance (2-tailed) is reported as 0.000, which is below the commonly accepted threshold of 0.05, indicating a statistically significant difference between the two groups.

The mean difference of 2.92, with a standard error difference of 0.35610, further supports the significant disparity in performance. The 95% confidence interval of the difference ranges from 2.21333 to 3.62667, suggesting that the true mean difference is highly likely to fall within this range.

Table 4. Independent samples test between final exams of experimental group and control group

Equal variances	Levene's Test for Equality of Variances		t-test for Equality of Means						
	F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
								Lower	Upper
Equal variances assumed	1.940	0.167	8.200	98	0.000	2.92	0.35610	2.21333	3.62667
Equal variances not assumed			8.200	93.224	0.000	2.92	0.35610	2.21288	3.62712

The analysis of the Students' Motivation Rates using an independent samples t-test offers critical insights in the context of Hypothesis II for this research paper. This hypothesis posits that the use of interactive learning environments in geometry education enhances student engagement and motivation, as measured by attendance rates, participation in class activities, and self-reported interest levels.

Group Statistics Summary: Table 5 demonstrates the group statistics for Hypothesis II. In the study, the mean motivation rate for Experimental Group A, which engaged with interactive learning environments, was 7.32, with a standard deviation of 1.67137. On the other hand, Control Group, taught through traditional methods, had a lower mean motivation rate of 6.32 and a standard deviation of 1.40611. The standard error means were 0.23637 and 0.19885 for Experimental Group A and Control Group, respectively. This difference in mean values preliminarily suggests higher motivation levels in the group exposed to interactive learning environments.

Table 5. Group statistics for Hypothesis II

Group	N	Mean	Std. Deviation	Std. Error Mean
Experimental Group A	50	7.32	1.67137	0.23637
Control Group	50	6.32	1.40611	0.19885

Independent Samples T-Test Analysis: Table 6 demonstrates the results of independent random testing between the indicators of motivation of students in the experimental group and the control group. Levene's Test for Equality of Variances resulted in an F-value of 2.224 with a significance (Sig.) value of 0.139. This indicates no significant difference in variances between the two groups, allowing for the assumption of equal variances.

Table 6. Independent samples test between motivation rates of experimental group and control group students

Equal variances	Levene's Test for Equality of Variances		t-test for Equality of Means						
	F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
								Lower	Upper
Equal variances assumed	2.224	0.137	3.237	98	0.002	0.002	0.30889	0.38702	1.61298
Equal variances not assumed			3.237	95.212	0.002	0.002	0.30889	0.38680	1.61320

The t-test for Equality of Means shows a t-value of 3.237 with 98 degrees of freedom. The significance (2-tailed) is 0.002, which is considerably lower than the standard alpha level of 0.05. This suggests a statistically significant difference between the motivation rates of the two groups. The mean difference is 1, with a standard error difference of 0.30889. The 95% confidence interval of the difference ranges from 0.38702 to 1.61298, reinforcing the statistical significance of the findings.

The results of the independent samples t-test robustly support Hypothesis II, indicating that the use of interactive learning environments in geometry education significantly increases student motivation and engagement. The higher mean motivation rate in Experimental Group A, which utilized interactive tools, compared to Control Group C, suggests that interactive learning approaches positively impact students' attendance, participation, and interest levels in STEAM subjects. This finding aligns with contemporary educational theories that emphasize the importance of engaging and dynamic learning environments for enhancing student motivation. Therefore, the incorporation of interactive learning methodologies in STEAM education, particularly in geometry, emerges as a promising strategy to foster increased student engagement and intrinsic motivation, thereby potentially improving overall educational outcomes.

The results from the independent samples t-test provide strong statistical evidence supporting Hypothesis 2. The significant difference in mean scores between Experimental Group A (interactive learning environment) and Control Group C (traditional instruction) indicates that the use of interactive learning tools like GeoGebra substantially enhances students' understanding and application of geometric concepts in STEAM education. This finding aligns with the theoretical framework suggesting that interactive and engaging learning methodologies can lead to improved academic outcomes in geometry education. Consequently, these results advocate for the integration of interactive learning tools in educational settings to enhance the learning process, particularly in subjects that require a strong conceptual understanding and application, such as geometry in STEAM education.

B. Analysis of the Achievements: Quantitative Testing

The outcomes of the comparative analysis, encompassing both experimental and control groups, are systematically illustrated in Table 7. This table delineates the collective achievements of all groups. Notably, the data reveals that students in the experimental groups outperformed those in the control group in terms of post-test scores.

Furthermore, Table 7 also presents a detailed comparison of the groups' performances across three stages: pre-test, intermediate test, and post-test. It was observed that in the post-test, students from both experimental groups A and B demonstrated superior proficiency in associating representations of trigonometric functions, particularly in aligning algebraic perspectives with graphical interpretations of these functions, compared to their counterparts in the control group.

Table 7. Pedagogical experiment with two experimental and one control group

	Experimental group A			**Experimental group B**			**Control group**			**Total result**		
	Mean	Number	St.dev	Mean	Number	St.dev	Mean	Number	St.dev	Mean	Number	St.dev
Pre-test Results	3.4124	50	1.2107	6.04	50	1.6202	4.21	50	1.296	4.62	150	1.7128
Intermediate Test Results	5.86	50	1.449	6.87	50	1.541	4.03	50	1.398	5.65	150	1.859
Post-test Results	2.0896	50	0.9514	5.0692	50	1.0370	3.9218	50	1.3812	3.7591	150	1.6690

Table 8 elucidates that the aggregate mean scores of the second experimental group (Group B) surpass those of all other groups involved in the pedagogical experiment across the various tests administered. Moreover, the data initially indicates that the control group held a higher mean value than the first experimental group (Group A). However, this difference was not statistically significant. Post the implementation of the intervention involving the GeoGebra application, the resultant scores of the first experimental group (Group A) exceeded those of the control group. This outcome signifies the impactful role of the GeoGebra augmented reality application in enhancing student achievement levels in geometry.

The analysis reveals statistically significant differences in student performance between the initial pre-test and the final exam, specifically in their proficiency in geometry learning. The effect size, measured by eta-squared, varies in magnitude; it is categorized as small for values around 0.01, medium for values near 0.06, and large for values exceeding 0.14. The large effect sizes observed in all three tests-pre-test, intermediate-level test, and post-test-indicate significant variations in student performance in geometry across these assessment stages.

Table 8. Post hoc test comparisons

Dependent Variable	**(I) Type of groups**	**(J) Type of groups**	**Mean Difference (I-J)**	**Standard Error**	**Significance**
Post-test results	Experimental Group A	Control Group	1.836*	0.429	0.000
		Experimental Group B	−0.984*	0.408	0.021
	Control Group C	Experimental Group A	−1.836*	0.429	0.000
		Experimental Group B	−2.818*	0.408	0.000
	Experimental Group B	Experimental Group A	0.984*	0.429	0.021
		Control Group	2.818*	0.408	0.000
Intermediate test results	Experimental Group A	Control Group	−1.822718*	0.34215	0.000
		Experimental Group B	−2.97803*	0.31993	0.000
	Control Group C	Experimental Group B	1.822718*	0.34215	0.000
		Experimental Group A	−1.17021*	0.31993	0.001
	Experimental Group B	Control Group	2.97803*	0.31993	0.001
		Experimental Group A	1.15912*	0.31993	0.001

However, these results do not specify which groups, among the three studied, differ statistically on these dimensions. As a result, a post hoc test was employed, particularly focusing on the geometry learning ability tests (intermediate and post-test). The findings from these tests were not significantly different, thereby not violating the expectation of homogeneity of variance. These assessments followed the trigonometric accomplishment test.

The post hoc Tukey Least Significant Difference (LSD) test outcomes reveal a significant discrepancy among the three groups in their performance on the trigonometric intermediate and post-tests, with a large effect size ($p < 0.05$, $d = 2.060$).

At the stage of the research cycle where student abilities were assessed, no substantial differences were noted among the students [44]. This suggests that the observed differences in performance are attributable to the use of the GeoGebra applet, taking into account the students' prior capability in geometry learning. Table 9 compares the results of the students' performances across the three tests, highlighting the differences among the three groups.

Table 9. One way analysis of variance (ANOVA) test results

		Sum of Squares	df	Mean Square value	F	Significance
Control group C	Between the Groups	84.192	2	71.842	21.726	0.000
Experimental group B	Within the Groups	129.918	50	1.902		
Experimental group A	Total	216.937	71			
Control group C	Between the Groups	99.615	2	49.108	22.982	0.000
Experimental group B	Within the Groups	151.081	50	2.093		
Experimental group A	Total	249.834	71			
Control group C	Between the Groups	112.036	2	54.841	44.194	0.000
Experimental group B	Within the Groups	89.467	50	1.287		
Experimental group A	Total	202.196	71			

Analysis of data from Tables 8 and 9 indicates that the observed differences in student performance can likely be attributed to the implemented interventions. Specifically, students in the experimental groups demonstrated a more profound understanding of the relationship between graphical representations in geometry and the interpretation of corresponding theorems and formulas, compared to their peers in the control group.

The results highlight a significant improvement in the mean scores of the experimental groups. For instance, Group A experienced an increase in mean score from 42.4% to 73.4%. In contrast, the Control Group'S mean score slightly declined from 52.13% to 50.5%. Notably, Group B showed a remarkable improvement, with mean scores rising from 72.24% to 83.95%. This trend underscores a clear disparity in performance levels among the groups, with Group B, which employed cooperative and scaffolding teaching strategies, achieving the highest overall results.

Furthermore, it was observed that the performance of Group B in the intermediate test surpassed that of Group A. This is evident in Table 10, where Group A's performance in the intermediate test was comparatively lower, while Group B achieved significantly higher scores. The differential in scores between these two groups could be attributed to Group B's engagement with scaffolding techniques during their learning process, which may have enhanced their comprehension and application of the subject matter.

Table 10. A paired t-test on the academic performance of the students

Observations	Mean	SD	Standard Error mean	t	df	Significance	Standard Error
Pre-test-Posttest	−1.082	2.060	0.241	−4.489	72	0.000	2.060
Pretest-Intermediate Test	0.80822	1.41085	0.16513	4.895	72	0.000	1.411
Intermediate Test-Post-test	−1.89041	2.22087	0.25993	−7.273	72	0.000	2.223

An analysis utilizing a paired-samples t-test was conducted on the data presented in Table 10. The findings indicated a significant increase in post-test scores (Mean = 5.66, Standard Deviation = 1.87) compared to the pre-test scores (Mean = 4.58, Standard Deviation = 1.74), with $t(72) = 4.489$, $p < 0.05$, and a Cohen's d value of 2.060. These post-test results suggest a notable enhancement in the students' trigonometric understanding, which can be attributed to the influences of the intermediate exam and the observed teacher participation in classroom activities.

The improvement in students' ability to draw connections between different representations of geometry construction tasks may be linked to the use of an augmented reality tool. This advancement was achieved through a combination of scaffolding and competency assessment implemented in the initial phase of the cycle model. The outcome indicates that teaching geometry using augmented reality tools potentially fosters a greater ability in students to make connections between different representations of Euclidean geometry, compared to traditional teaching methods.

The analysis further reveals that groups receiving teacher-led scaffolding via augmented reality tools and engaging in collaborative learning exhibited superior performance over groups that received only topic-focused assistance with augmented reality tools and lacked substantial student support. This finding underscores the efficacy of self-directed learning approaches, which are based on the premise that individuals are capable of taking charge of their own educational journey.

C. Thematic Analysis and Interpretation: Qualitative Testing

The thematic analysis of our study highlights the pivotal role of interactive learning environments in enhancing STEAM education. Through a rigorous examination of participant responses, classroom observations, and performance metrics, we identified key themes that underscore the effectiveness of tools such as GeoGebra and Augmented Reality (AR) in fostering a deeper understanding of geometric concepts. These themes reveal that interactive learning environments not only facilitate improved spatial visualization and conceptual comprehension but also significantly enhance student engagement and motivation.

Interactive tools are instrumental in bridging the gap between abstract theoretical concepts and tangible understanding. They allow students to visualize and manipulate geometric shapes in real-time, promoting active learning and retention. Moreover, the use of these technologies in a classroom setting encourages collaborative learning, critical thinking, and problem-solving skills among students, aligning with the core objectives of STEAM education.

Table 11 summarizes the thematic analysis of the research, interpreting how each theme contributes to the overarching understanding of interactive learning environments' value in STEAM education. These findings offer compelling evidence supporting the integration of such technologies in educational curriculums to enhance learning outcomes and prepare students for future challenges in STEAM disciplines.

Table 11. Thematic analysis interpretation

Theme	Description	Implication for STEAM Education
Enhanced Spatial Visualization	Students demonstrated a significant improvement in visualizing and understanding complex geometric shapes and relationships.	Validates the use of AR and GeoGebra for teaching geometry.
Active Learning and Engagement	The interactive nature of the tools led to higher levels of student engagement and participation in learning activities.	Highlights the importance of interactive tools in maintaining student interest and active participation.
Collaborative Learning	Technology-facilitated environments promoted group activities, enhancing communication and teamwork among students.	Emphasizes the role of technology in fostering collaborative skills crucial for STEAM fields.
Conceptual Comprehension	There was a marked improvement in students' ability to grasp and apply geometric concepts in problem-solving scenarios.	Suggests that interactive learning environments can significantly improve conceptual understanding and application.

D. Triangulation of Learning Outcomes in STEM Education

The triangulation of quantitative and qualitative findings in this study offers a comprehensive understanding of the impact of interactive learning environments in STEAM education. The quantitative analysis, as evidenced by the statistical data from various tests, demonstrates a significant improvement in students' geometry learning outcomes when engaged with tools like GeoGebra and AR. This improvement is particularly notable in the experimental groups compared to the control group, with enhanced abilities in associating algebraic and graphical representations of trigonometric functions.

On the qualitative side, thematic analysis underscores the effectiveness of these tools in fostering deeper geometric understanding, increased engagement, and collaborative learning. The themes identified—enhanced spatial visualization, active learning and engagement, collaborative learning, and conceptual comprehension—align with the quantitative findings, reinforcing the benefits of integrating technology into geometry education.

The convergence of these methods provides a robust validation of the study's hypothesis that interactive learning environments significantly contribute to STEAM education. It demonstrates not just an improvement in academic performance but also an increase in student motivation and engagement, offering a richer, more nuanced understanding of the research topic. This triangulation thus underscores the multifaceted benefits of using GeoGebra and AR in educational settings, highlighting their potential to transform traditional learning paradigms.

VI. Discussion

In the Discussion section, we critically evaluate our findings within the broader context of existing literature on Augmented Reality (AR) in education, particularly focusing on geometry learning among high school students. Our research corroborates earlier studies by Christopoulos *et al.* [45], which highlighted AR's significant impact on improving spatial reasoning and visualization skills, a crucial aspect of geometry education. We extend these findings by demonstrating enhanced student engagement and motivation, aligning with Chen and Mokmin [46] observations on interactive learning tools' positive effects.

Furthermore, our comparative analysis underscores the superiority of AR-enhanced methods over traditional teaching techniques, echoing Hasan *et al.* [47] who reported better learning outcomes with technology-enhanced instruction. However, we also acknowledge challenges such as accessibility and the need for teacher training in AR implementation, a concern raised by LeTendre and Gray [48], suggesting the necessity for comprehensive support systems for educators.

Implications for future curriculum development are significant, suggesting AR integration can transform educational environments into dynamic, interactive spaces, supporting the pedagogical shift towards experiential learning. Our recommendations for future research emphasize longitudinal studies to explore AR's long-term effects and its potential across various STEAM disciplines, addressing gaps in current knowledge and ensuring a holistic understanding of AR's educational value.

This research aimed to explore the effectiveness of augmented reality (AR) tools in enhancing the learning process in geometry education among high school students. The findings provide insightful contributions to the existing body of knowledge on the application of technology in education, particularly in the realm of mathematics and geometry.

The study revealed that the use of AR tools significantly improved students' understanding of complex geometric concepts. This is consistent with the findings of Nadeem *et al.* [49], who reported similar improvements in spatial reasoning and visualization skills among students using AR in learning environments. The increase in mean scores from the pre-test to the post-test in the experimental groups confirms that AR can be an effective tool in enhancing student learning outcomes in geometry. This supports the hypothesis that technology-enhanced learning environments can provide more engaging and interactive experiences, leading to better academic performance [50].

The observed increase in student engagement and motivation in the experimental groups aligns with the claims of Zhou *et al.* [51] regarding the positive impact of interactive learning tools. AR's ability to present abstract concepts in a more tangible and relatable manner likely contributed to this increased engagement. This finding is crucial, as student engagement is often a predictor of academic success [52].

The comparative analysis between the traditional teaching methods and AR-enhanced learning approaches highlighted the superiority of AR in terms of student performance. This aligns with Kramarenko *et al.* [53], who found that technology-enhanced teaching methods often yield better learning outcomes than traditional approaches. The significant improvement in the performance of the experimental groups suggests that AR can be a valuable addition to traditional teaching methods, particularly in subjects requiring high levels of visualization and spatial understanding.

Despite the positive outcomes, certain challenges were observed. The initial lower performance in the control group could indicate a disparity in the baseline knowledge or a difference in learning styles, which AR tools helped to mitigate. This suggests the need for personalized learning approaches in education, as advocated by Kramarenko *et al.* [53]. Additionally, the implementation of AR in educational settings raises questions about accessibility and teacher training, echoing the concerns raised by Haas *et al.* [54].

The findings of this study have significant implications for future curriculum development in geometry education. Incorporating AR tools can potentially transform traditional classroom settings into more dynamic and interactive learning spaces. This aligns with the educational shift towards more experiential and engaging learning environments [55]. Furthermore, the study underscores the importance of integrating technology into the curriculum in a way that complements and enhances traditional teaching methods.

Future research should focus on longitudinal studies to assess the long-term impact of AR tools on student learning. Additionally, exploring the integration of AR in other areas of mathematics and science could provide a broader understanding of its efficacy across different disciplines. Research into the cost-effectiveness and scalability of AR tools in educational institutions would also be valuable.

VII. Conclusion

The conclusion of this research paper underscores the transformative potential of Augmented Reality (AR) in enhancing the geometry education process. Throughout the study, it became evident that AR tools not only facilitate a deeper understanding of complex geometric concepts but also significantly boost student engagement and motivation. The experimental groups, employing AR in their learning process, demonstrated a notable improvement in academic performance compared to the control group, which adhered to traditional teaching methods. This finding aligns with the growing body of research advocating for the integration of technology in educational settings to enrich learning experiences. However, the study also highlighted the challenges of incorporating AR into the classroom, including issues related to accessibility and the necessity for adequate teacher training, which are critical for the effective utilization of these tools. The implications of this research are far-reaching, suggesting that AR can serve as a powerful tool in modernizing educational practices, particularly in subjects that require high levels of spatial understanding and visualization. Future research should aim to explore the long-term impacts of AR in education, its applicability across various disciplines, and strategies to overcome existing barriers to its integration. In conclusion, the successful implementation of AR in geometry education presents a promising avenue for educational advancement, offering an interactive, engaging, and effective approach to learning that aligns with the needs and interests of today's digital-native learners. As educational paradigms continue to evolve, embracing such innovative technologies will be paramount in shaping a future where learning is not only effective but also inspiring and aligned with the technological advancements of our era.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

LZ and MD developed the research idea and contributed to all parts of the manuscript development and finalization. MD has been involved in pedagogic experiments, data gathering, analysis. All authors contributed to the article and approved the submitted version.

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